Differences in Differences

Econ 140 Spring 2025, Section 9

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Syllabus/OH bCourses Website Feedback form (**Always open**) RStudio

Roadmap

1. Time Series
2. Diff-in-Diff
3. Making DiD more general: Panel Data estimation
4. Practice

Any questions?

...Remember – this is a safe space! Every question is useful!

Time Series

Why empirical microeconomists don't use time series

Spurious correlations here.

Unsolicited financial advice here.

A great idea to lose your tuition fees is here.

Time series topics

- Autoregressive and moving-average processes
- Serial correlation (of the errors)
- Unit roots
- Trends
- Detecting trend breaks
- ...

My prediction: Not the most important topic on the exam. But come to OH if you have questions or want to know more!

Practice

See here

Diff-in-Diff

Motivation for Differences in Differences

- In the first part of the course, we talked about comparison between groups or other units (cross-sectional data)
- But we have also seen some comparisons over time: time-series data
- Both of these have very big and obvious problems, but we can use them **together** using (panel data) in a powerful tool: Differences-in-differences!

How to get the causal effect of a treatment

In 2021, UC Berkeley offered free mental health coachings to students with pre-existing mental health issues. We want to evaluate the effect of this policy and we collect a depression score (0-10) for all students at UC Berkeley.

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated	4	5

We can do several comparisons:

How to get the causal effect of a treatment II

	2020	2022
Free Mental Health: Treated		6
No Free Mental Health: Untreated		5

We can do several comparisons:

- Comparison 1: Compare treated and non-treated group after the intervention

How to get the causal effect of a treatment II

	2020	2022
Free Mental Health: Treated		6
No Free Mental Health: Untreated		5

We can do several comparisons:

- Comparison 1: Compare treated and non-treated group after the intervention
- Estimated treatment effect? $6 - 5 = 1$

How to get the causal effect of a treatment II

	2020	2022
Free Mental Health: Treated		6
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We can do several comparisons:

- Comparison 1: Compare treated and non-treated group after the intervention
- Estimated treatment effect? $6 - 5 = 1$
- Problems?

How to get the causal effect of a treatment II

	2020	2022
Free Mental Health: Treated		6
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We can do several comparisons:

- Comparison 1: Compare treated and non-treated group after the intervention
- Estimated treatment effect? $6 - 5 = 1$
- Problems?
- **Identifying assumption**: The average difference between groups is due to the treatment only. Without the treatment, the average outcome of the treated group would have been equal to the average outcome of the control group.

How to get the causal effect of a treatment III

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated		

We can do several comparisons:

- Comparison 2: Compare treated group before and after the intervention

How to get the causal effect of a treatment III

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated		

We can do several comparisons:

- Comparison 2: Compare treated group before and after the intervention
- Estimated treatment effect? $6 - 6 = 0$

How to get the causal effect of a treatment III

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated		

We can do several comparisons:

- Comparison 2: Compare treated group before and after the intervention
- Estimated treatment effect? $6 - 6 = 0$
- Problems?

How to get the causal effect of a treatment III

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated		

We can do several comparisons:

- Comparison 2: Compare treated group before and after the intervention
- Estimated treatment effect? $6 - 6 = 0$
- Problems?
- Identifying assumption: The average difference across time is due to the treatment only. Without the treatment, the average outcome of the treated group would not have changed.

How to get the causal effect of a treatment: DiD

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated	4	5

We can do several comparisons:

- Comparison 3: Compare treated to untreated group, before and after the intervention. **Differences in differences!**

How to get the causal effect of a treatment: DiD

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated	4	5

We can do several comparisons:

- Comparison 3: Compare treated to untreated group, before and after the intervention. **Differences in differences!**
- Estimated treatment effect?
  $(6 - 6) - (5 - 4) = (6 - 5) - (6 - 4) = -1$

How to get the causal effect of a treatment: DiD

	2020	2022
Free Mental Health: Treated	6	6
No Free Mental Health: Untreated	4	5

We can do several comparisons:

- Comparison 3: Compare treated to untreated group, before and after the intervention. **Differences in differences!**
- Estimated treatment effect?
  $(6 - 6) - (5 - 4) = (6 - 5) - (6 - 4) = -1$
- Identifying assumption: Parallel trends: Without the treatment, the average increase in the outcome of the treated would have been the same as the average increase in the outcome of the untreated.

Sidenote

... We can NOT observe the identifying assumption! We can find evidence for or against it, but we can never be sure!

How to get the causal effect of a treatment: DiD

	2020	2022
Free Mental Health: Treated	6	6
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We can calculate the DiD estimate in two ways (here: superscripts stand for treatment/control group, subscripts denote unit i and time $t \in \{0, 1\}$)

$$DiD = E[\ \underbrace{\left(Y_{i1}^{T} - Y_{i0}^{T}\right)}_{\text{Change for treated}} - \underbrace{\left(Y_{i1}^{C} - Y_{i0}^{C}\right)}_{\text{Change for untreated}}\ ]$$

$$= E[\ \underbrace{\left(Y_{i1}^{T} - Y_{i1}^{C}\right)}_{\text{After-difference}} - \underbrace{\left(Y_{i0}^{T} - Y_{i0}^{C}\right)}_{\text{Before-difference}}\ ]$$

DiD and Potential Outcomes

Let us look at the second one in more detail:

$$
\begin{aligned}
DiD &= E[\left(Y_{i1}^T - Y_{i1}^C\right) - \left(Y_{i0}^T - Y_{i0}^C\right)] \\
&= \underbrace{\left(E[Y_{i1}^T] - E[Y_{i1}^C]\right)}_{\text{After-difference}} - \underbrace{\left(E[Y_{i0}^T] - E[Y_{i0}^C]\right)}_{\text{Before-difference}}
\end{aligned}
$$

Compare this with out formula for selection bias (in time period 1):

$$
\begin{aligned}
\Delta &= E[Y_{i1}^T] - E[Y_{i1}^C] \\
&= \underbrace{E[\mathrm{Y}_{i1}^T(1) - \mathrm{Y}_{i1}^T(0)]}_{\text{ATT}} + \underbrace{\left(E[\mathrm{Y}_{i1}^T(0)] - E[\mathrm{Y}_{i1}^C(0)]\right)}_{\text{Selection Bias}}
\end{aligned}
$$

DiD and Potential Outcomes

Let us look at the second one in more detail:

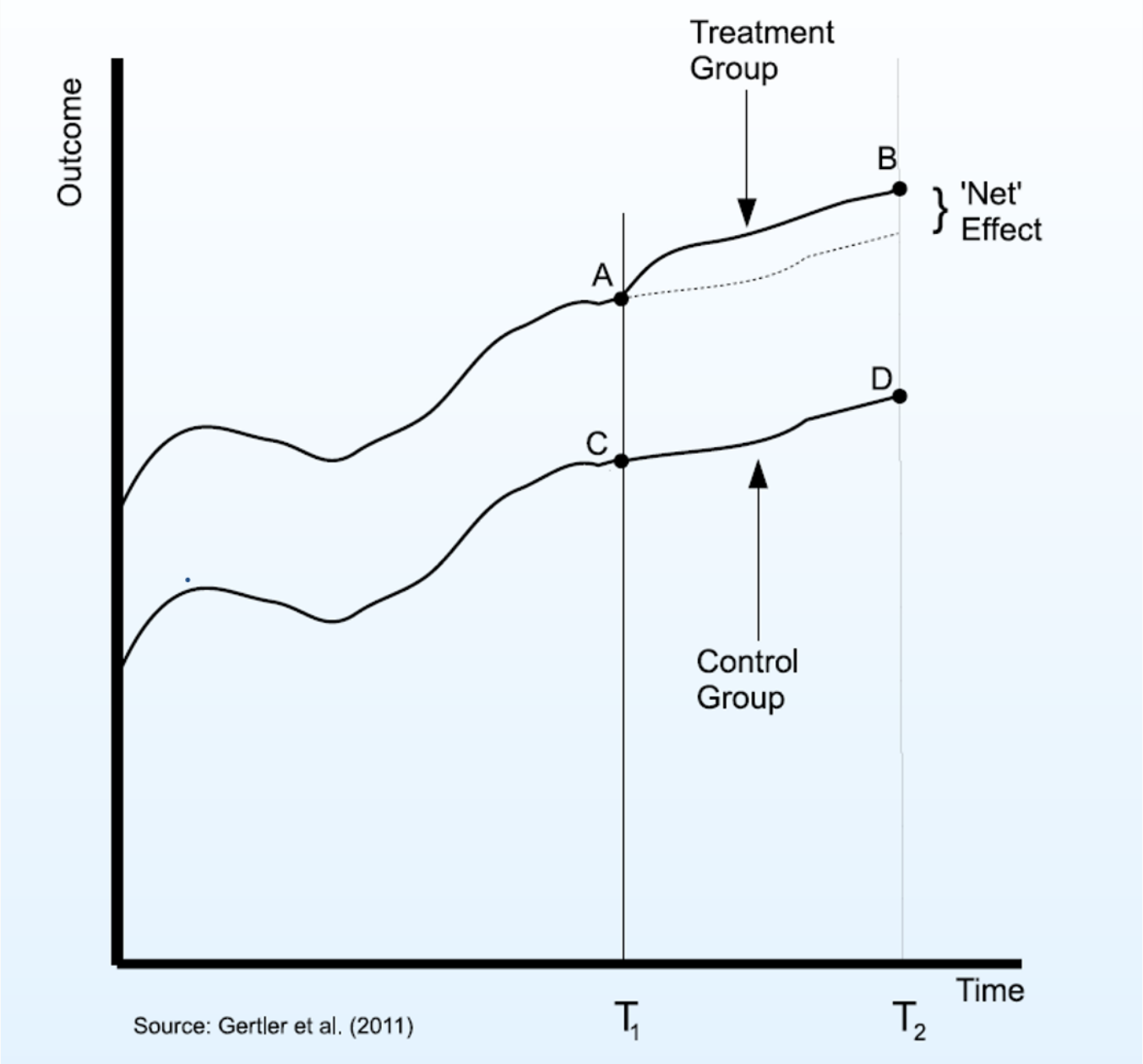
$$DiD = E[\left(Y_{i1}^T - Y_{i1}^C\right) - \left(Y_{i0}^T - Y_{i0}^C\right)]$$
$$= \underbrace{\left(E[Y_{i1}^T] - E[Y_{i1}^C]\right)}_{\text{After-difference}} - \underbrace{\left(E[Y_{i0}^T] - E[Y_{i0}^C]\right)}_{\text{Before-difference}}$$

Compare this with out formula for selection bias (at $t = 1$):

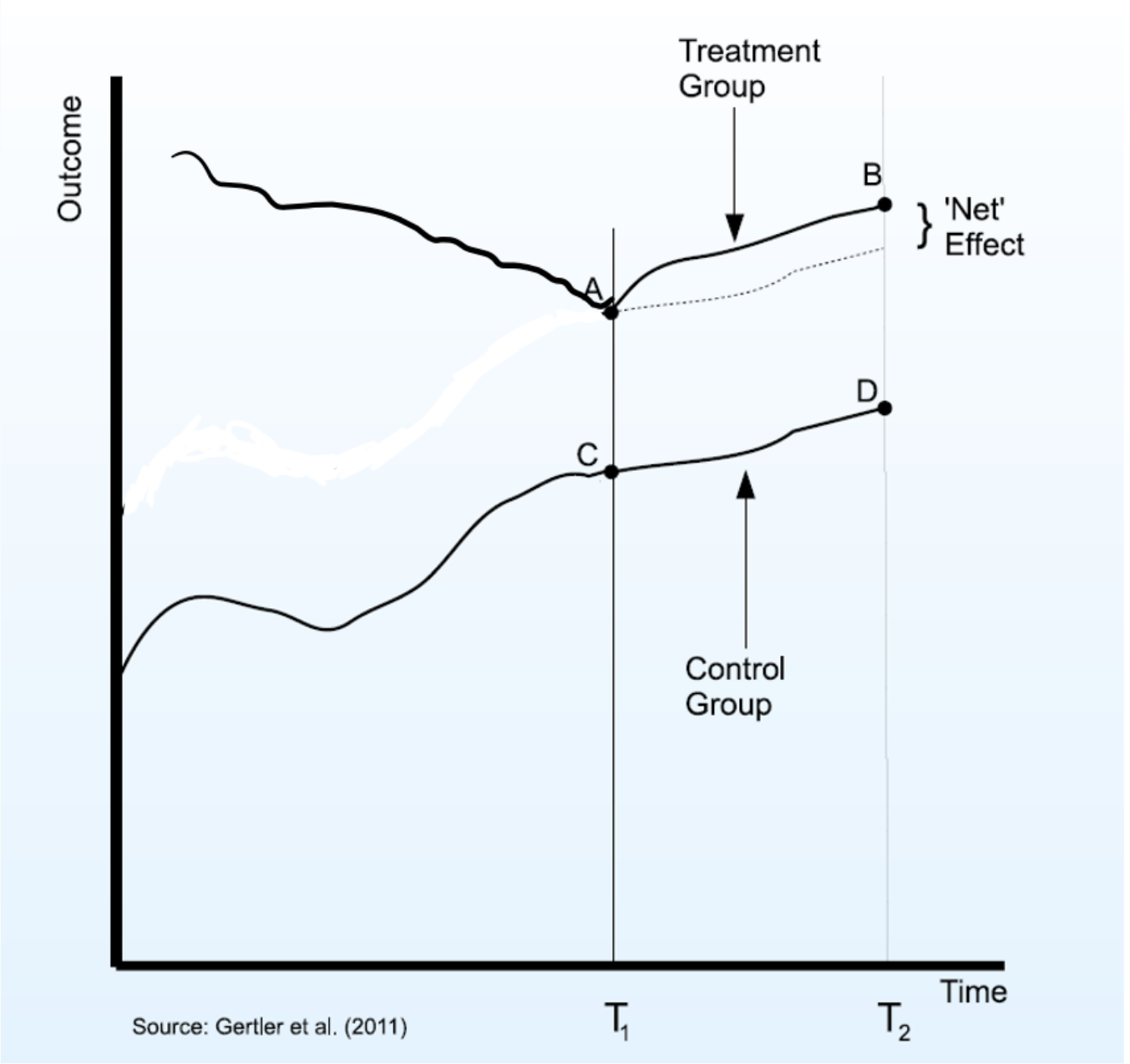
$$\Delta = E[Y_{i1}^T] - E[Y_{i1}^C]$$
$$= \underbrace{E[Y_{i1}^T(1) - Y_{i1}^T(0)]}_{\text{ATT}} + \underbrace{\left(E[Y_{i1}^T(0)] - E[Y_{i1}^C(0)]\right)}_{\text{Selection Bias}}$$

So DiD=ATT if the before-difference is equal to the selection bias in period 1: If the treatment group hadn't gotten the treatment (potential outcome = 0), the difference to the control group would have been the same as in period 0.

Parallel trends assumption



(Potential) violation of a parallel trends assumption



Estimating DiD with regressions

We can set up a simple linear regression to estimate a DiD model:

$$Y_{it} = \alpha + \beta \text{ Treated }_i + \gamma \text{ Post }_t + \delta \text{ Treated }_i \cdot \text{ Post }_t + u_{it}$$

	2020	2022
Free Mental Health: Treated		
No Free Mental Health: Untreated	α	

Estimating DiD with regressions

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	2020	2022
Free Mental Health: Treated		
No Free Mental Health: Untreated	α	$\alpha + \gamma$

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	2020	2022
Free Mental Health: Treated	$\alpha + \beta$	
No Free Mental Health: Untreated	α	$\alpha + \gamma$

Estimating DiD with regressions

We can set up a simple linear regression to estimate a DiD model:

$$Y_{it} = \alpha + \beta \text{ Treated }_i + \gamma \text{ Post }_t + \delta \text{ Treated }_i \cdot \text{ Post }_t + u_{it}$$

	2020	2022
Free Mental Health: Treated	$\alpha + \beta$	$\alpha + \beta + \gamma + \delta$
No Free Mental Health: Untreated	α	$\alpha + \gamma$

Estimating DiD with regressions

We can set up a simple linear regression model to get the DiD estimate:

$$Y_{it} = \alpha + \beta \text{ Treated }_i + \gamma \text{ Post }_t + \delta \text{ Treated }_i \cdot \text{ Post }_t + u_{it}$$

	2020	2022
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With our data, we would get:

- $\alpha = 4$
- $\beta = 2$
- $\gamma = 1$
- $\delta = -1$

Making DiD more general: Panel Data estimation

Limitations of the 2x2 DiD framework

In the simple 2x2 framework, we estimated:

$$Y_{it} = \alpha + \beta \text{ Treated }_i + \gamma \text{ Post }_t + \delta \text{ Treated }_i \cdot \text{ Post }_t + u_{it}$$

How can we extend this framework?

Intro to Panel Data i

A panel dataset may look like this:

	country	iso2c	iso3c	year	oda_per_cap	perc_below_pov_line	gdp_per_cap	inflation
316	Chile	CL	CHL	2015	3.0402700	0.4	13569.9478	4.34877353
317	Chile	CL	CHL	2016	9.8673522	*NA*	13785.6883	3.78619356
318	Chile	CL	CHL	2017	3.8277324	0.3	15045.5277	2.18271847
319	Chile	CL	CHL	2018	*NA*	*NA*	15795.7085	2.43488981
320	Chile	CL	CHL	2019	*NA*	*NA*	14631.9469	2.55754476
321	Chile	CL	CHL	2020	*NA*	0.7	13094.4595	3.04549085
322	Chile	CL	CHL	2021	*NA*	*NA*	16265.0960	4.52456838
323	China	CN	CHN	2015	−0.2219718	1.2	8016.4314	1.43702381
324	China	CN	CHN	2016	−0.5702808	0.8	8094.3634	2.00000182
325	China	CN	CHN	2017	−0.7090169	0.7	8816.9869	1.59313600
326	China	CN	CHN	2018	−0.5027731	0.4	9905.3420	2.07479040
327	China	CN	CHN	2019	−0.4323084	0.1	10143.8382	2.89923416
328	China	CN	CHN	2020	−0.4059315	*NA*	10408.6698	2.41942189
329	China	CN	CHN	2021	−0.4002945	*NA*	12556.3331	0.98101514
330	Colombia	CO	COL	2015	28.7760584	4.9	6228.4263	4.98983116
331	Colombia	CO	COL	2016	23.2331299	4.9	5938.4639	7.51346025
332	Colombia	CO	COL	2017	17.5871894	4.3	6450.3196	4.31431326
333	Colombia	CO	COL	2018	36.1416353	4.5	6782.0379	3.24056933
334	Colombia	CO	COL	2019	17.4085507	5.3	6438.0602	3.52301933

Intro to Panel Data ii

It may also look like this:

	Crime_2006	Crime_2007	Crime_2008	Crime_2009	Crime_2010	Crime_2011	Crime_2012	Crime_2013	Crime_2014	Crime_2015	Crime_2016	Crime_2017
New York	0,74075276	0,80312483	0,57854938	0,58480987	0,20097276	0,80826596	0,97070048	0,02437202	0,07831943	0,84572914	0,75749919	0,98948363
Los Angeles	0,81840795	0,63440595	0,75200179	0,45218138	0,54532883	0,0015679	0,39512876	0,41847691	0,66467241	0,01946605	0,55885996	0,27521867
Chicago	0,81682982	0,00089416	0,74575534	0,5521186	0,60461758	0,34694645	0,47874246	0,15925374	0,34667588	0,83944993	0,44825934	0,58746291
Houston	0,05885616	0,79349661	0,18033028	0,51317119	0,79642654	0,50269029	0,56157808	0,77173283	0,60797558	0,40461519	0,785332	0,6477959
Phoenix	0,2568483	0,28097691	0,82722175	0,11994755	0,884851	0,83800072	0,08978678	0,21613254	0,97306225	0,57291156	0,27341507	0,20641572
Philadelphia	0,86364857	0,30478317	0,07818559	0,82930161	0,28324875	0,60204952	0,4467311	0,1165021	0,15597767	0,36430802	0,38034388	0,37091781
San Antonio	0,98256294	0,13215208	0,55220195	0,25274349	0,22117177	0,63049228	0,74109619	0,17937274	0,50914785	0,68773119	0,17854372	0,50634586
San Diego	0,26052609	0,90551179	0,96145068	0,04932108	0,55859179	0,57753748	0,38254621	0,36728436	0,39429268	0,40312699	0,55506545	0,51665312
Dallas	0,85045327	0,33482019	0,15361924	0,74231968	0,55762722	0,05057305	0,57960707	0,71774087	0,63565644	0,04305477	0,06451316	0,27719973
San Jose	0,75452737	0,21462214	0,24820183	0,2727016	0,00127307	0,36822105	0,32304996	0,9116018	0,86006177	0,05904204	0,1124391	0,83247218
Austin	0,45398495	0,27082977	0,89920786	0,82480029	0,52877813	0,9452614	0,82578514	0,828743	0,20606871	0,4439492	0,38857348	0,8154967
Jacksonville	0,18638302	0,66744344	0,38392491	0,83800597	0,30374371	0,16785163	0,53614039	0,15558203	0,14260844	0,51748284	0,07054862	0,70962828
Fort Worth	0,13254183	0,78355507	0,83321159	0,33460775	0,00772913	0,78127289	0,61026409	0,25552213	0,27373821	0,78588972	0,80784366	0,1307398
Columbus	0,21457262	0,59542339	0,65347292	0,16532587	0,35580394	0,7113844	0,72036005	0,39323372	0,73022336	0,32649945	0,95442814	0,44923343
Indianapolis	0,97993863	0,2017115	0,58883837	0,54914541	0,22495244	0,58047281	0,01984661	0,88891756	0,34920826	0,326581	0,11407985	0,76592867
Charlotte	0,13043704	0,44544929	0,13581143	0,32176217	0,06451777	0,60708331	0,24805805	0,91489751	0,39053247	0,36409354	0,94249222	0,12812588

Interpreting FE regressions

Suppose we collect yearly data on firms' profits and the gender of the firms' CEO, for a sample of 1,000 firms over 20 years.

We can estimate this model with firm and year fixed effects:

$$\text{Profits}_{it} = \beta_0 + \beta_1 \text{Female CEO}_{it} + \alpha_i + \delta_t + \varepsilon_{it}$$

How do we interpret β_1?

Interpreting FE regressions

Suppose we collect yearly data on firms' profits and the gender of the firms' CEO, for a sample of 1,000 firms over 20 years.

We can estimate this model with firm and year fixed effects:

$$\text{Profits}_{it} = \beta_0 + \beta_1 \text{Female CEO}_{it} + \alpha_i + \delta_t + \varepsilon_{it}$$

How do we interpret β_1?

The same way as in an OLS regression!

β_1 describes how having a female CEO is associated with profits, holding constant all firm-specific factors that do not change over time, and all time-specific factors that affect all firms equally. β_1 is the effect within each firm (and year!), controlling for the firm-specific and year-specific fixed effects.

Visualization

See separate gif!

Limitations of fixed effects

- Imagine we want to estimate the effect of annual income on happiness and we have a panel following 1,000 individuals over 10 years. You think there are many omitted variables and include individual-level fixed effects. You are **also** interested in the effect of parental income on happiness. What's the problem?

Limitations of fixed effects

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- We want to estimate the effect of democracy on economic growth in a panel of 180 countries over 60 years. We can include country-fixed effects to account for any effect coming from different countries having a different climate, terrain, access to the sea, culture, etc. Are you satisfied with this approach?

Limitations of fixed effects

- Imagine we want to estimate the effect of annual income on happiness and we have a panel following 1,000 individuals over 10 years. You think there are many omitted variables and include individual-level fixed effects. You are **also** interested in the effect of parental income on happiness. What's the problem?
- We want to estimate the effect of democracy on economic growth in a panel of 180 countries over 60 years. We can include country-fixed effects to account for any effect coming from different countries having a different climate, terrain, access to the sea, culture, etc. Are you satisfied with this approach?
- We want to estimate the effect of education on wages and think of using individual-level fixed effects. What can go wrong?

Any questions?

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Practice

Practice

See here