Method	Estimation details	Identifying assumption	Formal assumption	Threats to identification	How to verify identifying assumption?	Which effect is estimated?
OLS with controls	$Y_i = \beta_0 + \boldsymbol{\beta_1} X_i + \gamma C_i + \epsilon_i$ The slope coefficient, $\boldsymbol{\beta_1}$, tells us how much we expect Y to change for a one-unit change in X, conditional on controls C.	Conditional on controls, in the absence of the treatment, the outcome of the treated group would have been the same as the outcome of the control group	$E[\epsilon_i \mid X_i] = Cov(X_{i_i} \epsilon_i) = 0$ $E[Y_i(0) \mid T_i = 1] = E[Y_i(0) \mid T_i = 0]$	OVB, classical measurement error, reverse causality, misspecfication of functional form, systematic measurement error, sample selection: Anything that causes the error to be correlated with X!	• Add set of sensible controls, see how coefficient changes • Come up with plausible stories	ATT
RCT with perfect compliance	Estimate with OLS	Treatment is, conditionally, randomly assigned – same as in OLS.	Same as in OLS	Same as in OLS	Balance table: At baseline, check whether there are significant differences between treatment and control.	ATT = ATE
IV	• First stage: $X_i = \alpha_0 + \alpha_1 Z_i + \pi C_i + u_i$ • Reduced form: $Y_i = \gamma_0 + \gamma_1 Z_i + \eta C_i + v_i$ • Target regression: $Y_i = \beta_0 + \boldsymbol{\beta_1} X_i + \delta C_i + \epsilon_i$ IV coefficient is first stage divided by reduced form. Estimated with 2SLS, esp. for multiple instruments.	We are comparing units with different levels of the treatment X, where the only difference we use is coming from differences in the instrument Z. 1. No defiers / monotonicity 2. Instrument relevance 3. Instrument exogeneity 4. Exclusion restriction	1. $X_i(1) \geq X_i(0)$ 2. $Cov(X_i, Z_i) \neq 0$ 3. $Cov(Z_i, \epsilon_i) = 0$ 4. $Cov(Z_i, \epsilon_i) = 0$ Mathematically speaking, exogeneity and exclusion restriction are the same.	Violations of IV assumptions.	1. Has to be assumed. 2. F-test>10, with one instrument for one endogenous variable: square of t-statistic 3. Balance table 4. Use brain	LATE LATE = ATT if there are no always-takers
RCT with imperfect compliance	Use assignment to treatment group as instrument for actual treatment status. Estimate with IV. The Reduced form estimates ITT.	Same as IV	Same as IV	Same as in IV & RCT	Same as IV & RCT	ITT (reduced form) or: LATE (= ATT if there are no always-takers)
DiD	In 2x2 case, take differences in table. Regression implementation: $Y_{it} = \alpha_0 + \alpha_1 T_i + \delta A_t + \boldsymbol{\beta} T_i \times A_t + \epsilon_{it}$, i.e. use an indicator for treatment group, an indicator for after time period, and interaction. $\boldsymbol{\beta}$ is the DiD coefficient.	Parallel Trends: In the absence of the treatment, the outcome of the treatment group would have changed by the same amount as the outcome of the control group. DiD removes any bias from the fact that: Individuals are inherently different from each other, or specific time periods are inherently different from each other.	$E[Y_{i0}^T] - E[Y_{i0}^C] = E[Y_{i0}^T(0)] - E[Y_{i0}^C(0)]$ $Cov(X_{it}, \epsilon_{it}) = 0$	Violations of parallel trends: 1. Something changed over time that affected treated and control group differently 2. The effect of X on Y changed over time, in way that is correlated with X	• Look at pre-trends: If they are not parallel, there is a problem • But we cannot prove this, ergo: Use brain	ATT, but estimated within groups, not between
Fixed Effects	Essentially the same as in DiD (for two-way fixed effects): $Y_{it} = \alpha_0 + \alpha_i + \delta_t + \boldsymbol{\beta} T_{it} + \epsilon_{it}$ α_i are unit fixed effects (dummy variables), δ_t are time fixed effects. Estimate with a seriers of dummy variables, or by demeaning. Can only estimate effects for time-varying treatments.	No time-changing confounders, which is the same thing as DiD, or as OLS with controls, but only using time-changing confounders.	$Cov(X_{it}, \epsilon_{it}) = 0$ which is the same assumption as in DiD.	Time-changing confounders: Did anything happen over time that affected T and C differently? Did the effect of X on Y change over time in a way that differs across units?	• Same as DiD • Also the same as OLS with controls, but look at time-changing confounders	ATT, But estimated within groups, not between
Sharp RD	$Y_{it} = \alpha_0 + f(X_i) + \boldsymbol{\beta} T_i + \epsilon_i$ Where $f(X_i)$ is a flexible functional form – linear or polynomial – of the running variable that determines the treatment. T_i is an indicator for whether i is above the threshold. Need to choose bandwitdth, the "window" we use to estimate the size of the jump.	There is no jump, expect our jump, at the jump, i.e., at the threshold, the only thing that jumps is the probability of treatment, i.e. 1. Probability of getting treatment jumps at threshold 2. No other pre-determined variable jumps at the threshold 3. The only way that being above the threshold affects the outcome is by changing the probability of getting the treatment. No strategic behavior / bunching around threshold!		• Other baseline variables jump at threshold • Strategic behavior and bunching around threshold (information) • Threshold is also used for other purposes	• Run RD (or make plot) with other baseline variables • Use brain: If eligibility for program X and for program Y use the same threshold, then not identified • Look at plots to identify bunching/strategic behavior around threshold	ATT
Fuzzy RD is just IV	Estimate with IV/2SLS. Use "being above the threshold" as instrument for the treatment. In all specifications, control for running variable.	Same as Sharp RD, which is same as IV (verify that the two sets of conditions are exactly the same!)		• Same as sharp RD = same as IV	• Same as sharp RD • Same as IV	LATE (= ATT if there are no always-takers, i.e. if no one below the threshold gets the treatment)

Terminology

OLS: Ordinary Least Squares
RCT: Randomized controlled Trial
DiD: Differences-in-Differences
IV: Instrumental variables
RD: Regression discontinuity

ATE: Average Treatment Effect
ATT = ToT: Average Treatment Effect for the Treated
LATE: Local Average Treatment Effect = Treatment Effect for the Compliers

Numbers in parentheses denote **potential outcomes.**
Subscripts with numbers denote **time periods**.

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